Probing extra dimensions with higher dimensional black hole analogues?

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Abstract

We propose that extra dimensions might be detected with higher dimensional analogues of black holes. The usual 4-dimensional acoustic(sonic)black hole metric is extended to arbitrary dimensions. The absorption cross-section of Hawking radiation on the brane and in the bulk are calculated in the semiclassical approximation.
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1 Introduction

A lot of interest in recent years has been raised for field theories where the standard model of high-energy physics is assumed to live on a 3-brane embedded in a larger space-time, while only the gravitational fields are in contrast usually considered to live in the whole spacetime [1,2,3,4]. Arkani-Hamed *et al* [2], proposed a new framework for solving the hierarchy problem which does not rely on supersymmetry or techi-color. The novelty in this idea was that the traditional picture of Planck-length-sized additional spacelike dimensions ($l_p \simeq 10^{-33}$cm) was abandoned, and the extra dimensions could have a size as large as 1 mm. The fact that we do not see experimental signs of the extra dimensions despite that the compactification scale of the extra dimensions $\mu_c \sim 1/V_n^{1/n}$ would have to be much smaller than the weak scale, implies that only gravity can propagate in the extra-dimensional spacetime and all ordinary matter: electromagnetic, weak and strong forces, is restricted to live on a (3+1) dimensional hypersurfaces, a 3-brane.

The presence of extra dimensions in brane world gravity models will

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inevitably change the properties and physics of black holes. In brane-world scenario, the production cross-section for black holes is greatly enhanced. Detectable signals of black holes might be found via Hawking evaporation to brane-localized modes. If the scale of quantum gravity is near a Tev, the Large Hadron Collider (LHC) will become a "black hole factory" [5]. The correlations between the black hole mass and its temperature, deduced from the energy spectrum of the decay products, can test Hawking's evaporation law. The absorption cross-section (and also grey body factors) of black holes in different physical scenario have been investigated by several authors[6,7,8,9,10]. The emission rate of 4-dimensional acoustic black holes for S wave has been studied in Ref.[11]. In this paper, we would like to investigate the absorption cross-section of higher dimensional acoustic black holes.

The remarkable work of Unruh, in 1981, developed a way of mapping certain aspects of black holes in supersonic flows and pointed out that propagation of sound in a fluid or gas turning supersonic [12], is similar to the propagation of a scalar field close to a black hole, and thus experimental investigation of the Hawking radiation is possible. From then on, several candidates have been considered for the experimental test of the analogue of black holes [13].

In the present study, we extend 4-dimensional acoustic black hole metric into arbitrary dimensional space-time in Section 2. Then following the methods of Ref.[9], we calculate the corresponding absorption probabilities and energy emission rates of such black hole analogues. In section 3, we present the calculation of bulk emission in $(4+n)$-dimensions for $l \geq 0$ case by using the low energy perturbation method. Section 4 performs the calculation of brane-localized scalar emission, where the brane is embedded in a $(4+n)$-dimensional bulk. We give the conclusions in Section 5.

2 Metric of higher dimensional black hole analogues

Since the dynamics of effective black hole analogues metric are not described by the laws of gravity (i.e. Einstein equations) in general, to extend the four dimensional effective metric we should start with the equations of fluid dynamics. We consider one kind of fluids which is permitted to propagate in $(4+n)$-dimensional space-time. The fluid might be condensate states of gravitons or some higher-dimensional scalar particles. In the following of our discussions, we simply neglect quantum effects of such special quantum fluids and assume they obey the classical fluids equation (Euler equation) in the low energy approximation. The fundamental equations of fluid dynamics in $(4+n)$-dimensional flat spacetime are the equation of continuity

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0, \tag{1}$$

and Euler's equations

$$\rho\frac{d\vec{v}}{dt} \equiv \rho[\partial_t\vec{v} + (\vec{v}\cdot\nabla)\vec{v}] = \vec{F}, \tag{2}$$

where $\nabla = \hat{e}_x\frac{\partial}{\partial x} + \hat{e}_y\frac{\partial}{\partial y} + \hat{e}_z\frac{\partial}{\partial z} + ... + \hat{e}_n\frac{\partial}{\partial n}$, and $\vec{F} = -\nabla p - \rho\nabla\phi$. Here ϕ denotes the Newtonian gravitational potential. It is also assumed that the fluid to be irrotational and inviscid, which imply $\nabla\times\vec{v} = 0$. These equations can be linearized in the vicinity of some mean flow solution with $\rho = \rho_0 + \varepsilon\rho_1 + O(\varepsilon^2)$, $p = p_0 + \varepsilon p_1 + O(\varepsilon^2)$,$\varphi = \varphi_0 + \varepsilon\varphi_1 + O(\varepsilon^2)$, redefining the fields as $\nabla h = \frac{\nabla p}{\rho}$, and $\vec{v} = \nabla\varphi$. One can finally obtain the wave equation [14],

$$-\partial_t\left[\frac{\partial\rho}{\partial p}\rho_0(\partial_t\varphi_1 + \vec{v}_0\cdot\nabla\varphi)\right] + \nabla\cdot\left[\rho_0\nabla\varphi_1 - \frac{\partial\rho}{\partial p}\rho_0\vec{v}_0(\partial_t\varphi_1 + \vec{v}_0\cdot\nabla\varphi_1)\right] = 0 \tag{3}$$

The above equation is identified with a massless scalar field equation describing the sound wave in the curved spacetime background

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi_1) = 0 \tag{4}$$

with the background metric, $g_{\mu\nu} = \left(\frac{\rho_0}{c}\right)^{\frac{2}{n+2}}\begin{pmatrix} -c^2 + v_0^2 & -v_0^i \\ -v_0^j & \delta_{ij}\end{pmatrix}$, which is a $(n+4)\times(n+4)$ matrix, where the local speed of sound is defined by $c^2 \equiv \frac{\partial p}{\partial\rho}$, and $i, j = 1...n$. In spherical coordinates, assuming $v_r \neq 0$, $v_{\theta_1} = v_{\theta_2} = ... = 0$, we then have

$$ds^2 = \left(\frac{\rho_0}{c}\right)^{\frac{2}{n+2}}\left[-c^2(1-\frac{v_r^2}{c^2})d\tau^2 + (1-\frac{v_r^2}{c^2})^{-1}dr^2 + r^2 d\Omega_{n+2}^2\right], \tag{5}$$

which is similar to the n-dimensional Schwarzschild black hole metric [15],

$$ds^2 = \left[-\mathrm{C}^2\left(1-\frac{r_H^{n+1}}{\mathrm{C}^2 r^{n+1}}\right)dt^2 + \left(1-\frac{r_H^{n+1}}{\mathrm{C}^2 r^{n+1}}\right)^{-1}dr^2 + r^2 d\Omega_{n+2}^2\right], \tag{6}$$

where C is the light velocity and r_H is the event horizon radius. The properties of higher dimensional Schwarzschild black holes in brane-world scenario have been discussed in Ref.[16]. However, the properties of acoustic black holes do not necessarily relate to the outside geometry of space-time since the dynamics of acoustic black holes do not obey Einstein equation. In Ref.[17], we have discussed black hole analogues in brane-world scenario under some assumptions and found the properties of acoustic black holes in brane-world scenario are

similar to real black holes. If ρ is time and position independent, the continuity equation $\nabla \cdot \vec{v} = 0$ then implies that $v_r \propto \frac{1}{r^{n+2}}$. Because of the barotropic assumption, ρ is position independent implying the pressure p and the speed of sound c are also position independent. We can define a normalization constant r_0 and set $v_r = \mathcal{C}/r^{n+2}$ with $\mathcal{C} = c r_0^{n+2}$. r_0 is a parameter which can be determined by experiments. The metric can be rewritten as

$$d\tilde{s}^2 = \left[-c^2 \left(1 - \frac{{r_0}^{2n+4}}{r^{2n+4}} \right) d\tau^2 + \left(1 - \frac{{r_0}^{2n+4}}{r^{2n+4}} \right)^{-1} dr^2 + r^2 d\Omega_{n+2}^2 \right], \quad (7)$$

where $d\tilde{s}^2 = (\frac{c}{\rho_0})^{\frac{2}{n+2}} ds^2$. The temperature of an acoustic(sonic) black hole is given by $T = \frac{\hbar}{2\pi k} \left| \frac{\partial v_r}{\partial r} \right|_{v_r = c}$, where $v_r = c$ marks the exact position of the event horizon. For 4-dimensional cases, $v_r = c\frac{r_0'^2}{r^2}$, we have the numerical expression,

$$T = 6 \times 10^{-7} K [c/300\text{m/sec}][1 \text{ mm}/r_0']. \quad (8)$$

For (4+n)-dimensional cases, $v_r = c\frac{r_0^{n+2}}{r^{n+2}}$, the temperature can then be written as

$$T = 3(n+2) \times 10^{-7} K [c/300\text{m/sec}][1\text{mm}/r_0]. \quad (9)$$

It is clear from the above equations that unless c is very large (i.e. to be the velocity of light $c = 3 \times 10^8$ m/sec), the experimental verification of above acoustic Hawking temperature will be rather difficult. However, the higher dimensional black hole analogue metric presents us an alternative way to detect extra dimensions other than in LHC. By detecting signals via the acoustic black hole's evaporation to brane-localized modes, one can in principle determine the exact dimensions of space-time. However, since the Standard Model particles must to be located to an ordinary 4-dimensional spacetime, extra dimensions can only be probed through the gravitational force. The continuity equation $\nabla \cdot \vec{v} = 0$ in higher dimensions should only apply to gravitational waves or some unknown scalar particles.

3 Bulk scalar emission: S wave and $l \geq 0$

In this section and the next section, we shall calculate the decay rate of higher dimensional acoustic black holes from the near-horizon low energy dynamics, which is expected to give an experimental suggestion of detecting the thermal radiation.

The radiation of higher dimensional acoustic black holes is usually described as thermal spectrum in character with a temperature T_{4+n}. The energy emitted per unit time (power spectrum) by gravity-wave black hole analogues

for a higher number of dimensions can be given by

$$\frac{dE(\omega)}{dt} = \sum_l \sigma_{l,n}(\omega) \frac{\omega}{exp(\omega/T_{4+n}) - 1} \frac{d^{n+3}k}{(2\pi)^{n+3}}, \tag{10}$$

where l is the angular momentum quantum number and $\mid k \mid= \omega$ for massless particles. However, considering the nontrivial metric in the region exterior to the horizon, there exists an effective potential barrier in this exterior region. This potential barrier backscatters a part of the outgoing radiation back into the black hole. Thus the original blackbody radiation is modified by a frequency dependent filtering function $\sigma_l(\omega)$, caused by the gravitational potential of the black hole, which is called the "greybody factor". Greybody factors are important theoretically and experimentally in that they depend on the number of extra dimensions and encode information on the near-horizon structure of black holes and can be used to identify a black hole event.

For S wave bulk scalar emission of the $(4+n)$-dimensional gravity-wave black hole analogues, the calculation is similar to that of higher dimensional Schwarzschild cases. The calculations do not depend on the concrete form of the metric. In fact, it has been proved that all spherically symmetric black holes, regardless of the theory in which they arise, the low energy cross section for massless minimally coupled scalars is always the area of the horizon [18]. Following the method of Ref. [10], we can obtain the spherical wave absorption probability,

$$\mid \mathcal{A}(\omega) \mid^2 = \frac{2(\omega r_0)^{n+2} sin[\pi(n+1)/2]}{2^n(n+1)} \frac{\Gamma(\frac{1-n}{2})}{\Gamma(\frac{3+n}{2})} \tag{11}$$

The greybody factor can be computed by first evaluating the absorption probability, $\mid \mathcal{A}(\omega) \mid^2$, from the ratio of the in-going flux at the future horizon to the incoming flux from past infinity with boundary condition that there is no outgoing flux at the horizon, and then using the generalized $(4+n)$-dimensional optical theorem relation [19]

$$\sigma_l(\omega) = \frac{2^n \pi^{(n+1)/2} \Gamma(\frac{n+1}{2})}{n! \omega^{n+2}} \frac{(2l+n+1)(l+n)!}{l!} \mid \mathcal{A}(\omega) \mid^2 \tag{12}$$

between the absorption cross section $\sigma_l(\omega)$ and the absorption probability $\mid \mathcal{A}(\omega) \mid^2$ for the l-th partial waves.

We will then derive the scalar decay modes which are not spherically symmetric, $l \neq 0$. We start with the $4+n$-dimensional black hole analogue by rewriting Eq.(7) in the following form,

$$ds^2 = -h(r)dt^2 + h(r)^{-1}dr^2 + r^2 d\Omega^2_{n+2}, \tag{13}$$

where $h(r) = 1 - \left(\frac{r_0}{r}\right)^{2n+4}$ and we have assumed $\rho_0 = c = 1$. The scalar wave equation in this background is separable if we make the ansatz $\phi(t, r, \theta, \varphi) = e^{-i\omega t} R_{wl}(r) Y_l(\Omega)$, where $Y_l(\Omega)$ are now the $(3+n)$-spatial dimensional spherical

harmonic functions. We can then obtain the radial part equation by substituting the above ansatz into the Eq.(4), which reads,

$$\frac{h(r)}{r^{n+2}}\frac{d}{dr}\left[h(r)r^{n+2}\frac{dR}{dr}\right]+\left[\omega^2-\frac{h(r)l(l+n+1)}{r^2}\right]R=0. \tag{14}$$

The idea of [7] is to solve this equation approximately in three regions: near-horizon regions, far field regions and intermediate regions, and match the solution across the boundaries of the regions. We will use the same method here, but simply solve the radial equation in near-horizon and far field regions, and assume that keep only the lowest order terms in ω in each region is enough. From the change of variables $0 \le r \to h \le 1$, we can write the scalar field equation Eq.(14) in the form,

$$\begin{aligned} &h(1-h)\frac{d^2R}{dh^2}+\left(1-\frac{3n+7}{2n+4}h\right)\frac{dR}{dh}\\ &+\left[\frac{\omega^2 r_0^2}{(2n+4)^2h(1-h)}-\frac{l(l+n+1)}{(2n+4)^2(1-h)}\right]R=0, \end{aligned} \tag{15}$$

where we set $(\omega r)^2$ to be $(\omega r_0)^2$ near the horizon. By redefining $R(h)=h^\alpha(1-h)^\beta F(h)$ and removing singularities at $h=0$ and $h=1$, the above equation can be reduced to a hypergeometric equation with $a=\alpha+\beta+\frac{n+3}{2n+4}$, $b=\alpha+\beta$ and $c=2\alpha+1$, where,

$$\alpha_\pm=\pm\frac{i\omega r_0}{2n+4}, \qquad \beta=\frac{1}{2}\pm\frac{1}{2(n+2)}\sqrt{(l+n+2)^2-4\omega^2 r_0^2}. \tag{16}$$

Thus, we have,

$$h(1-h)\frac{d^2F}{dh^2}+\left[c-(1+a+b)h\right]\frac{dF}{dh}-abF=0, \tag{17}$$

which has a solution the hypergeometric function $F(a,b,c;h)$ [20]. The criterion for the convergence of the hypergeometric function demands that $Re(c-a-b)>0$, which force us to choose $\beta=\beta_-$. Then, the general solution of Eq.(**??**) is,

$$\begin{aligned} R_{NH}(h)=\,&A_-h^{\alpha_\pm}(1-h)^\beta F(a,b,c;h)\\ &+A_+h^{\pm\alpha_\pm}(1-h)^\beta F(a-c+1,b-c+1,2-c;h) \end{aligned} \tag{18}$$

Expanding the above solution in the near-horizon region in the limit $r\to r_0$, or $h\to 0$, and choosing $\alpha=\alpha_-$, we obtain the result,

$$R_{NH}(h) = (\frac{r_0}{r})^{2\beta(n+2)} \left[A_- e^{(-i\omega r_0^{n+2} y)} + A_+ e^{(i\omega r_0^{n+2} y)}\right], \tag{19}$$

where y is defined by $y = \frac{\ln h(r)}{r_0^{n+1}(n+1)}$. To calculate the greybody factor, we must impose the boundary condition that near the horizon the solution is purely ingoing and then we set $A_+ = 0$ [9].

By using the $h \to (1-h)$ transformation of hypergeometric functions,

$$F(a,b,c;h) = \frac{\Gamma(n)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F(a,b;a+b-c+1;1-h)$$
$$+(1-h)^{c-a-b} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F(c-a;c-b,c-a-b+1;1-h), \tag{20}$$

the near-horizon solution (18) expanded in terms of $1-h$ is given by,

$$R_{NH}(h) = A_- h^{\alpha} \Bigg[(1-h)^{\beta} \frac{\Gamma(1+2\alpha)\Gamma(1-2\beta-\frac{n+3}{2n+4})}{\Gamma(1+\alpha-\beta-\frac{n+3}{2n+4})\Gamma(1+\alpha-\beta)} F(a,b,a+b-c+1;1-h)$$
$$+(1-h)^{1-\beta-\frac{n+3}{2n+4}} \frac{\Gamma(1+2\alpha)\Gamma(2\beta+\frac{n+3}{2n+4}-1)}{\Gamma(\alpha+\beta+\frac{n+3}{2n+4})\Gamma(\alpha+\beta)} F(c-a,c-b,c-a-b+1;1-h)\Bigg], \tag{21}$$

where A_+ has been set to vanish. Expanding the above expression in the limit $h \to 1$, we get,

$$R_{NH}(h) = A_- (\frac{r}{r_0})^{l} \frac{\Gamma(1+2\alpha)\Gamma(1-2\beta-\frac{n+3}{2n+4})}{\Gamma(1+\alpha-\beta-\frac{n+3}{2n+4})\Gamma(1+\alpha-\beta)}$$
$$+(\frac{r_0}{r})^{l+n+1} \frac{\Gamma(1+2\alpha)\Gamma(2\beta+\frac{n+3}{2n+4}-1)}{\Gamma(\alpha+\beta+\frac{n+3}{2n+4})\Gamma(\alpha+\beta)}. \tag{22}$$

The derivation of the far-field zone which is defined by $r \gg r_0$. In this limit, $h(r) \simeq 1$ and, by setting $R(r) = f(r)/r^{(n+1)/2}$, Eq.(14) can be rewritten as

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} + \left[\omega^2 - \frac{(2l+n+1)^2}{4r^2}\right] f = 0, \tag{23}$$

which is a $(l+\frac{n+1}{2})$-th order Bessel equation. The solution of the above equation are the Bessel functions

$$R_{FF} = \frac{B_+}{r^{(n+1)/2}} J_{l+\frac{n+1}{2}}(\omega r) + \frac{B_-}{r^{(n+1)/2}} Y_{l+\frac{n+1}{2}}(\omega r), \tag{24}$$

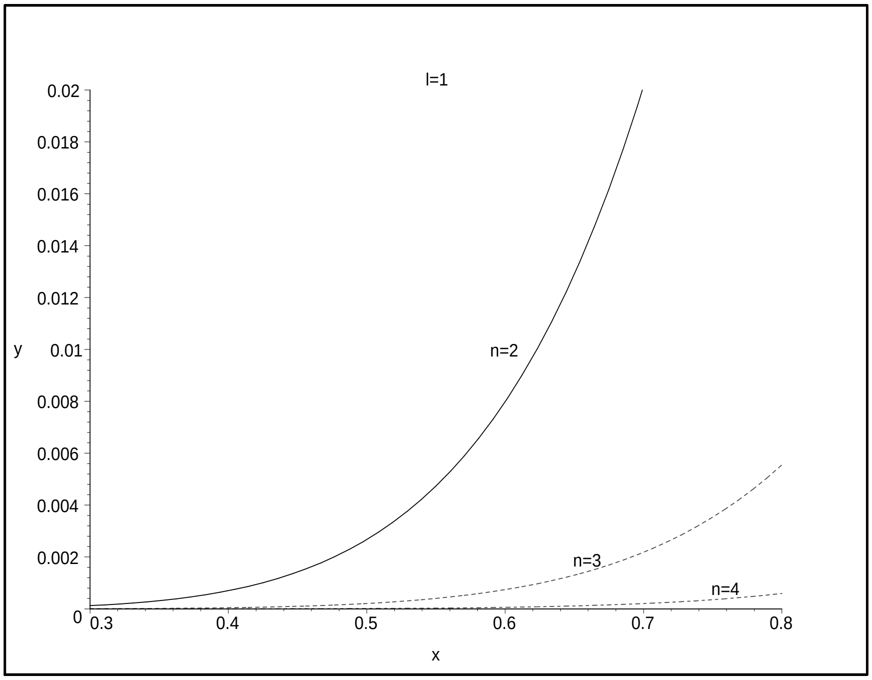

Figure 1. Analytical results for the absorption probability for a $(4+n)$-dimensional bulk scalar field for $l=1$, where $x=\omega r_0$ and $y=\mid \mathcal{A}\mid^2$.

where $J_{l+\frac{n+1}{2}}(\omega r)$ and $Y_{l+\frac{n+1}{2}}(\omega r)$ are the Bessel functions of the first and second kind, respectively. For small $\omega r \ll 1$, we can expand the above formula into,

$$R_{FF} \simeq \frac{B_+ r^l}{\Gamma(l+\frac{n+3}{2})}\left(\frac{\omega}{2}\right)^{l+\frac{n+1}{2}} - \frac{B_-}{r^{l+n+1}}\left(\frac{2}{\omega}\right)^{l+\frac{n+1}{2}} \frac{\Gamma(l+\frac{n+1}{2})}{\pi} \tag{25}$$

In order to match the solutions across the boundaries of near-horizon field and far-field zone, we need to rewrite the near-horizon solution in terms of $(1-h)$, before expanding the solution in the limit $r \gg r_0$. Matching the two solutions Eqs.(22) and (25), we obtain the ratio,

$$\frac{B_+}{B_-} \approx -\left(\frac{2}{\omega r_0}\right)^{2l+n+1} \frac{\Gamma(l+\frac{n+1}{2})^2(l+\frac{n+1}{2})\Gamma(1-2\beta-\frac{n+3}{2n+4})\Gamma(\alpha+\beta)\Gamma(\alpha+\beta+\frac{n+3}{2n+4})}{\pi\Gamma(1+\alpha-\beta)\Gamma(1+\alpha-\beta-\frac{n+3}{2n+4})\Gamma(2\beta+\frac{n+3}{2n+4}-1)}. \tag{26}$$

The ingoing and outgoing waves of Eq.(25) can be decomposed by introducing the redefinition of amplitudes. We expand Eq.(25) in the limit $r \to \infty$, and then we obtain,

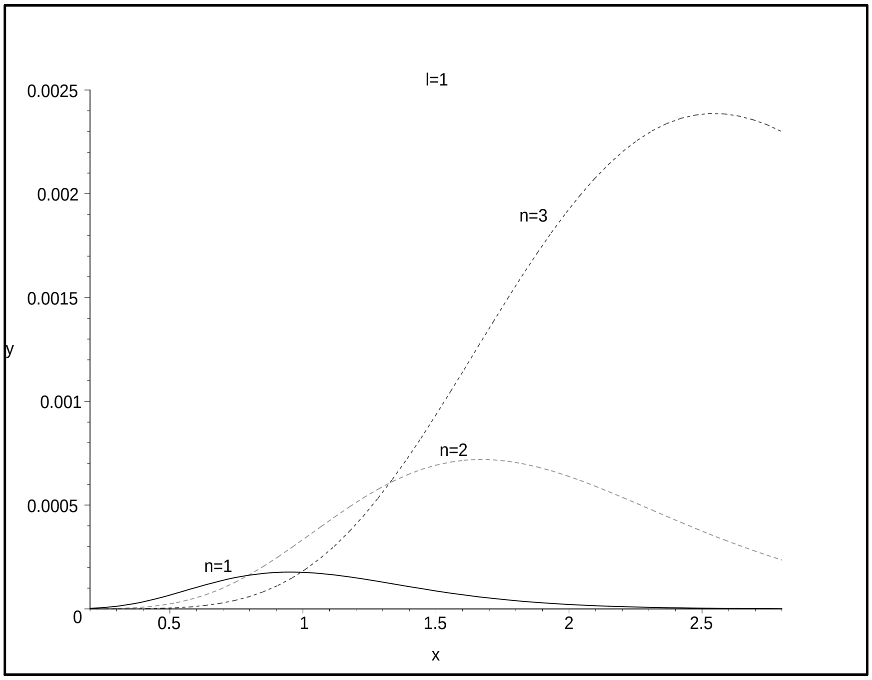

Figure 2. Analytical results for the energy rates for scalars from a $(4+n)$-dimensional black hole in the bulk for $l = 1$, where $x = \omega r_0$ and $y = \frac{d^2E}{dtd\omega}[r_0^{-1}]$.

$$R^{(\infty)} = A_{in}^{(\infty)} \frac{e^{-i\omega r}}{\sqrt{r(n+2)}} + A_{out}^{(\infty)} \frac{e^{i\omega r}}{\sqrt{r(n+2)}}, \tag{27}$$

where $A_{in}^{(\infty)}$ and $A_{out}^{(\infty)}$ is defined by,

$$\begin{aligned} A_{in}^{(\infty)} &= \frac{B_+ + iB_-}{\sqrt{2\pi\omega}} e^{i\pi(l+\frac{n}{2}+1)/2}, \\ A_{out}^{(\infty)} &= \frac{B_+ - iB_-}{\sqrt{2\pi\omega}} e^{-i\pi(l+\frac{n}{2}+1)/2} \end{aligned} \tag{28}$$

The reflection coefficient $\mathcal{R}$ is defined as the ratio of the outgoing amplitude over the incoming amplitude at infinity,

$$\mathcal{R} = \left| \frac{A_{out}^{(\infty)}}{A_{in}^{(\infty)}} \right|^2. \tag{29}$$

The absorption probability can be written, in terms of $B = B_+/B_-$, as

$$| \mathcal{A}(\omega) |^2 = 1 - | \mathcal{R} |^2 = \frac{2i(B^* - B)}{BB^* + i(B^* - B) + 1}. \tag{30}$$

In the limit of $wr_0 \ll 1$ and $BB^* \gg i(B^* - B) \gg 1$, the absorption probability can approximately be written as

$$| \mathcal{A}(\omega) |^2 \simeq \frac{8\pi(2n+5-2l)}{(l+\frac{n+1}{2})(2l-1)^2} \times$$
$$(\frac{\omega r_0}{2})^{2l+n+2} \frac{\Gamma(\frac{3}{4}+\frac{l}{2n+4})^2 \Gamma(\frac{n}{4(n+2)}+\frac{l}{2n+4})^2}{\Gamma(l+\frac{n+1}{2})^2 \Gamma(\frac{2l-1}{2n+4})^2} \tag{31}$$

From Figure 1, we can see that if we fix the angular momentum number and vary only the number of extra dimensions, the absorption probability decreases as n increases, since the expansions of $| \mathcal{A} |^2$ is in powers of $\omega r_0 \ll 1$. Thus, $| \mathcal{A} |^2$ should become more and more suppressed as n increases. The same behavior is observed if we fix instead n and vary l.

According to Figure 2, the emission rate of scalar fields in the bulk is enhanced as the number of extra dimensions increases. This is caused by the increase of the temperature of gravity-wave black hole analogues, which finally overcome the decreases in the value of the greybody factor and causes the enhancement of the emission rate with n at high energies.

4 Brane-localized scalar emission for $l \geq 0$

If the acoustic black hole is formed from matter on the brane, the scalar field is confined on the 3-brane embedded in a $(4+n)$-dimensional space-time. The induced metric on the brane will be,

$$ds^2 = -h(r)dt^2 + h(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{32}$$

where $h(r) = 1-(\frac{r_0}{r})^{2(n+2)}$. on the brane, the event horizon is still at $r = r_0$ and its area is $A_4 = 4\pi r_0^2$. This induced acoustic metric on the brane is certainly not the 4-dimensional acoustic geometry. The calculation of Hawking radiation relies on mainly on properties of the horizon, such as its surface gravity. We shall calculate the absorption cross-section of the brane-localized scalar field, which according to Ref.[21], most of the energy radiated by black holes goes into modes on the brane. Using the separation of variables, $\phi(t, r, \theta, \varphi) = e^{-i\omega t} R_{wl}(r) Y_l(\Omega)$, where $Y_l(\Omega)$ are now the usual three-dimensional spherical harmonic functions, the radial equation of Eq.(4) is written as,

$$\frac{h(r)}{r^2}\frac{d}{dr}\left[h(r)r^2\frac{dR}{dr}\right] + \left[\omega^2 - \frac{h(r)}{r^2}l(l+1)\right] R = 0 \tag{33}$$

Similar to the discussions in section 3, we solve this equation in two regions: the near-horizon region and far-field region. Our starting point is the Klein-Gordon equation under the near-horizon metric background. In terms of h, the radial differential equation now takes the form,

$$
\begin{aligned}
&h(1-h)\frac{d^2R}{dh^2}+\left[1-\frac{4n+7}{2(n+2)}\right]\frac{dR}{dh}\\
&+\left[\frac{\omega^2r^2}{(2n+4)^2h(1-h)}-\frac{l(l+1)}{(2n+4)^2(1-h)}\right]R=0
\end{aligned}
\tag{34}
$$

If we further define $R(h)=h^{\alpha}(1-h)^{\beta}F(h)$ to remove singularities at points $h=0$ and $h=1$, the above equation assumes the standard form of a hypergeometric equation

$$
h(1-h)\frac{d^2F}{dh^2}+[c-(1+a+b)h]\frac{dF}{dh}-abF=0,
\tag{35}
$$

with indices $a=\alpha+\beta+\frac{2n+3}{2n+4}$,$b=\alpha+\beta$ and $c=1+2\alpha$, where $\alpha_{\pm}=\frac{i\omega r}{2n+2}$ and $\beta=\frac{1}{4(n+2)}(1\pm\sqrt{(2l+1)^2-4\omega^2r^2})$. The criterion for the convergence of the horizon and imposing the boundary condition that only incoming waves exist near $r\simeq r_0$, one can find that $A_+=0$ for $\alpha=\alpha_-$. To express the form of the solution for small h, we express the $F(h)$ in terms of $1-h$, by using the hypergeometric relation Eq.(20). Thus after expanding $R_{NH}(h)$ for $r\gg r_0$, we find that the desired solution for near-horizon region is

$$
\begin{aligned}
&R_{NH}(h)\simeq A_-(\frac{r}{r_0})^l\frac{\Gamma(1+2\alpha)\Gamma(1-2\beta-\frac{2n+3}{2n+4})}{\Gamma(1+\alpha-\beta-\frac{2n+3}{2n+4})\Gamma(1+\alpha+\beta)}\\
&+A_-(\frac{r_0}{r})^{l+1}\frac{\Gamma(1+2\alpha)\Gamma(2\beta+\frac{2n+3}{2n+4}-1)}{\Gamma(\alpha-\beta+\frac{2n+3}{2n+4})\Gamma(\alpha+\beta)}
\end{aligned}
\tag{36}
$$

For the far-field region which is defined by $r\gg r_0$, noting that $h(r)\to 1$ and setting $R(r)=f(r)/r^{1/2}$, we rewrite Eq.(33) as,

$$
\frac{d^2f}{dr^2}+\frac{1}{r}\frac{df}{dr}+\left[\omega^2-\frac{(2l+1)^2}{4r^2}\right]f=0.
\tag{37}
$$

The far field region solution can be expressed in terms of the Bessel function $J_{l+1/2}(\omega r)$ and Neumann function $Y_{l+1/2}(\omega r)$,

$$
R_{FF}=\frac{B_+}{r^{1/2}}J_{l+\frac{1}{2}}(\omega r)+\frac{B_-}{r^{1/2}}Y_{l+\frac{1}{2}}(\omega r),
\tag{38}
$$

At this stage, we expand the general solution Eq.(38), in the low energy limit $\omega r\ll 1$ and find that,

$$
R_{FF}\simeq\frac{B_+r^l}{\Gamma(l+3/2)}\left(\frac{\omega}{2}\right)^{l+1/2}-\frac{B_-}{r^{l+1}}\left(\frac{2}{\omega}\right)^{l+1/2}\frac{\Gamma(l+1/2)}{\pi}
\tag{39}
$$

Matching the solution Eq.(36) with (39), we obtain the ratio,

$$\frac{B_+}{B_-} = -\left(\frac{2}{\omega r_0}\right)^{2l+1}$$
$$\frac{\Gamma(l+\frac{1}{2})^2(l+\frac{1}{2})\Gamma(1-2\beta-\frac{2n+3}{2n+4})\Gamma(\alpha+\beta)\Gamma(\alpha+\beta+\frac{2n+3}{2n+4})}{\pi\Gamma(1+\alpha-\beta)\Gamma(1+\alpha-\beta-\frac{2n+3}{2n+4})\Gamma(2\beta+\frac{2n+3}{2n+4}-1)} \tag{40}$$

We can now compute the absorption probability $\mid \mathcal{A}(\omega) \mid^2$ in the low energy limit $\omega r_0 \ll 1$, which goes as,

$$\mid \mathcal{A}(\omega) \mid^2 = \frac{16\pi}{(2l+1)^2}\left(\frac{\omega r_0}{2}\right)^{2l+2}\frac{\Gamma(\frac{l+1}{2n+4})^2\Gamma(1+\frac{l}{2n+4})^2}{\Gamma(l+\frac{1}{2})^2\Gamma(1+\frac{2l+1}{2n+4})^2} \tag{41}$$

Figure 3 demonstrates that if we keep n fixed and varying l, the absorption probability decreases as l increases. The dominant term becomes more and more suppressed by extra powers of wr_0 and its numerical coefficients also decreases. If we fix instead l and vary n, a different behavior from the one observed in the case with a bulk scalar field emerges, that is to say the leading term remains the same since it is n-independent. Figure 4 is to compare the results of the absorption probability derived in the case of a brane-localized ($n > 0$) scalar field with a purely 4-dimensional ($n = 0$) scalar field. For higher partial waves ($l > 0$), the value of the absorption probability in the case of a brane-localized ($n > 0$) scalar field is larger than the one for a purely 4-dimensional ($n = 0$) field, while for $l = 0$ they are the same. Figure 5 depicts the behavior of the energy emission rates for particles with the angular number $l = 1$ in the low and intermediate energy regime. The figure shows that the energy and the number of particles, emitted per unit time and energy interval is strongly enhanced, as n increases since the temperature of the black hole is given by the relation $T_{4+n} \propto (n+2)/2\pi r_0$, which indicates that for fixed r_0, the temperature of the gravity-wave black hole analogues increases as n increases. This means the energy of the emission particles also increases.

5 Conclusions

In summary, we have extended the 4-dimensional acoustic black hole metric to higher dimensions that is similar to higher dimensional Schwarzschild metric in form but is not exactly any of the standard geometries typically considered in general relativity. The fluids here have been assumed to fill all the spacial dimensions including extra dimensions. We emphasize that the higher dimensional acoustic black holes discussed above is just a model, which may help us understand the physics beyond the standard model and are falsifiable

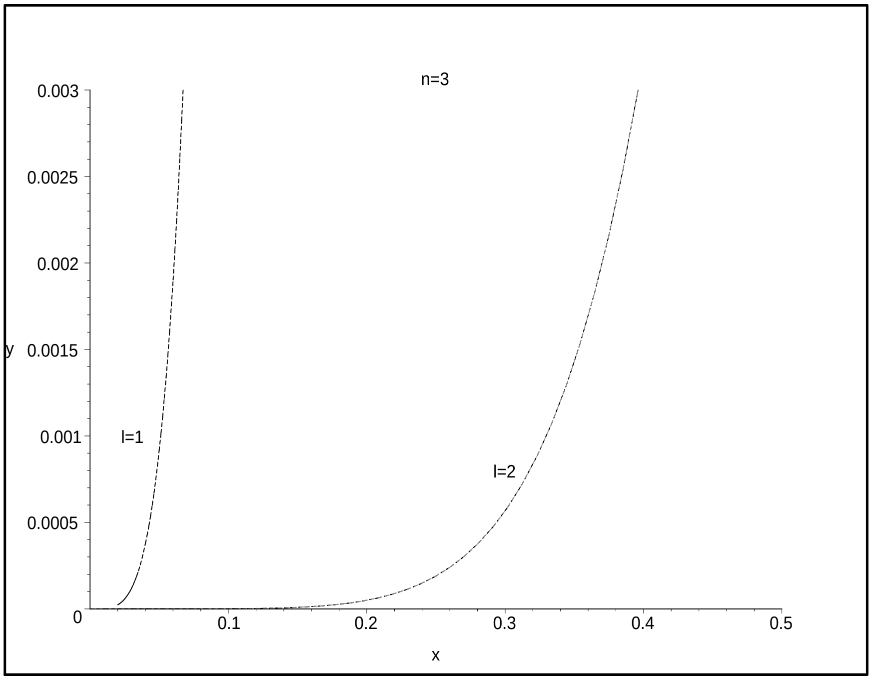

Figure 3. Analytical results for the absorption probability for a $(4+n)$ black holes analogues on the brane for $n = 3$, where $x = \omega r_0$ and $y =| \mathcal{A} |^2$.

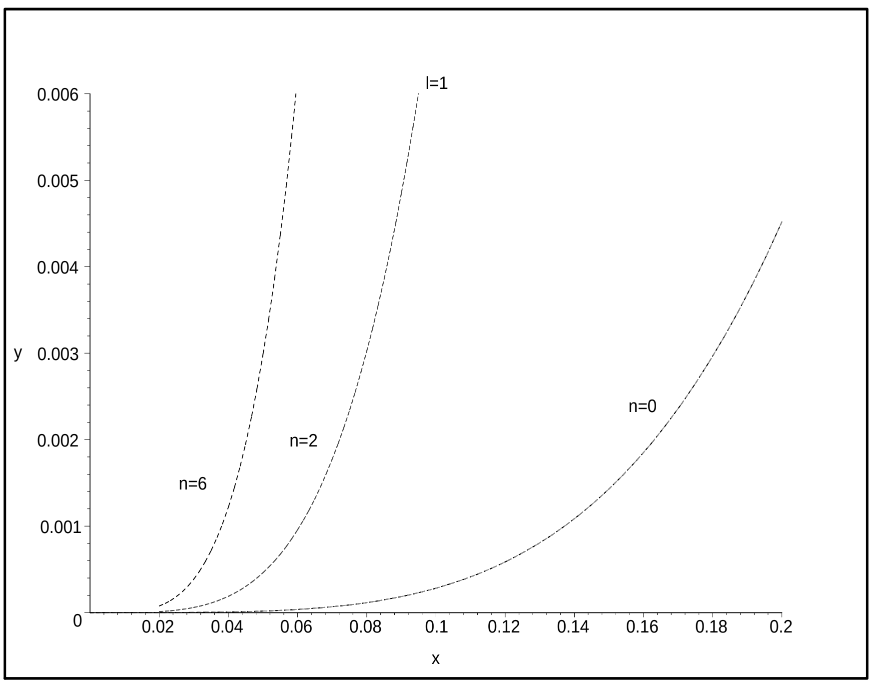

Figure 4. Analytical results for the absorption probability for a $(4+n)$ black holes analogues on the brane for $l = 3$, where $x = \omega r_0$ and $y =| \mathcal{A} |^2$.

by the future experiments. Although it might be difficult to detect the Hawking effects of these analogues in experiments since the Hawking temperature in fact is very low, they provide us an otherwise way to probe extra dimensions.

The scalar emission of Hawking particles in both $(4+n)$-dimensional bulk

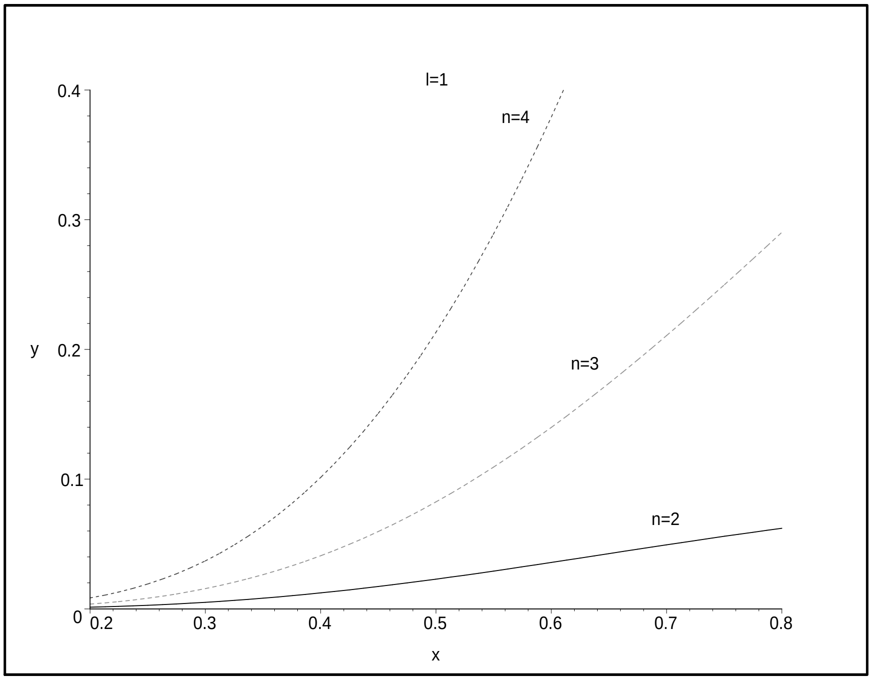

Figure 5. Analytical results for the energy rates for scalars from a $(4 + n)$-dimensional black hole on the brane for $l = 1$, where $x = \omega r_0$ and $y = \frac{d^2E}{dtd\omega}[r_0^{-1}]$.

scalar field and 4-dimensional brane-localized scalar field were studied respectively in a higher dimensional gravity-wave black hole analogues background. The amplitude probability in a bulk scalar field was obtained that allows one to find its dependence on the number of extra dimensions n and the angular momentum number l. We found that if we fix the angular momentum number and vary only the number of extra dimensions, the absorption probability decreases as n increases, and $\mid \mathcal{A} \mid^2$ should become more and more suppressed as n increases. The same behavior is observed if we fix instead n and vary l. The case in which the scalar field is confined on a 3-brane in a higher dimensional spacetime background is also discussed. The resulting absorption probability depends only on the angular momentum number through $(\omega r_0)^{2l+2}$. If we keep n fixed and varying l, the absorption probability decreases as l increases. But, in both bulk and brane-localized cases, the energy emission rates are enhanced as n increases since the temperature of the gravity-wave black hole analogues increases as n increases.

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